Comparison of Scaling Methods for Short-Duration Rainfall Frequency Analysis

Karin Xuereb
Senior Meteorologist, Climate and Water Division, Bureau of Meteorology, Melbourne, Australia
E-mail: k.xuereb@bom.gov.au

Janice Green
IFD Revision Project Manager, Climate and Water Division, Bureau of Meteorology, Melbourne, Australia
E-mail: j.green@bom.gov.au

Abstract

Estimates of design rainfall from the Australian Rainfall and Runoff Intensity-Frequency-Duration (IFD) revision are to be provided for durations from one minute up to at least seven days. At the very short durations the record length is relatively short and spatial coverage is sparse leading to difficulties in developing sub-hourly design rainfall estimates directly from the data. Bayesian Generalised Least Squares Regression (BGLSR) provides quantiles to a duration of one hour and supplements pluviometer data. For durations shorter than one hour, scaling methods are explored that use information at the longer durations where data availability is longer temporally and wider spatially to infer sub-hourly design rainfall quantile estimates.

Four methods are explored: ratios of rainfall annual maxima, ratios of quantiles, simple scaling and extending the BGLSR down to one minute. The methods have been validated to establish the most appropriate method to adopt for the IFD revision. Practitioners will be able to have design rainfall estimates available to a duration of one minute that are based on all the data available and the best techniques in short duration rainfall frequency estimation. It is found that the relatively simple method of average ratios of annual maxima performs best.

1. INTRODUCTION

Revised estimates of design rainfall for durations from 1 minute to at least seven days will be presented by the revision of Intensity-Frequency-Duration design rainfall. The network of daily rainfall gauges is adequate both in terms of spatial coverage and length of record for deriving reliable estimates of design rainfall for durations of one day and longer. For sub-daily durations, however, the network of continuous rainfall gauges is relatively sparse and insufficient for adequate mapping of design rainfall. In order to supplement the data availability at sub-daily time scales, Bayesian Generalised Least Squares Regression (BGLSR) (Haddad *et al*, 2010) has been used to infer rainfall statistics to a duration of one hour using data at the daily durations as well as a set of predictors such as elevation, slope, aspect and distance from the coast. The method has been tested and found to perform well down to one-hour duration.

The availability of data at a resolution of one minute is even sparser and record lengths even shorter than that for five minute time intervals and longer. It only became possible to obtain data at such short time intervals with the installation of Tipping Bucket Rain Gauges (TBRGs) in the mid 1990s. This means that record lengths for one-minute data span at most 20 years and the spatial coverage of TBRGs is also quite sparse.

The BGLSR will supplement pluviometer data at daily rainfall sites down to one-hour duration. In this study, techniques are explored on how to derive design rainfall estimates for standard durations of 1, 5, 10, 15 and 30 minutes using 1-hour data.

Techniques that have been adopted in the past have used the available sub-hourly data and derived scaling factors between the short and long durations. Perica *et al* (2011) used averaged ratios of annual maxima at the required 15 and 30 minutes to the corresponding 60-minute annual maxima. Regionally averaged ratios were then applied to sites with 60-minute data to generate sub-hourly pseudo data.

Bonnin *et al* (2004), evaluated ratios of *n*-minute quantiles (where *n* is the required short duration) to 60-minute quantiles.

Using the assumption of scale-invariance, Menabde *et al* (1999) showed that rainfall intensities at different time resolutions are related through a simple scaling factor. A more complex multiscaling approach relates the small scale and large scale properties of rainfall are by a function involving several parameters (Mandapaka *et al*, 2009).

In this study, the methods of 1) ratios of annual maxima, 2) ratios of quantiles, 3) simple scaling and 4) BGLSR have been tested to estimate sub-hourly rainfall at 1-, 5-, 10-, 15- and 30-minute durations from 60-minute rainfall data. The methods will then be validated by applying the Kolmogorov-Smirnov test by comparing the cumulative distribution functions of the modeled sub-hourly values against sub-hourly values from pluviometer data. The best performing method will then be applied to extend the pseudo hourly data from the BGLSR down to the 1-minute duration.

Bureau of Meteorology TBRG data was used to derive estimates at the 1-minute duration. For the 5-, 10-, 15- and 30-minute durations, Bureau of Meteorology Dines pluviograph data as well as TBRG data was used.

2. METHOD

2.1. Ratios of Annual Maxima

The ratios of the 1-minute annual maxima (AM) to the 60-minute AM were calculated for data having TBRG data and then averaged. Average ratios of 5-, 10-, 15-, 30-minute annual maxima to 60 minute AM were prepared using Dines and TBRG pluviometer data. The ratios do not vary much over distances of, say 500 hundred kilometers but there is a latitudinal variation with ratios tending to be smaller in the north than in the south of the country at the 1-minute duration (Figure 1).

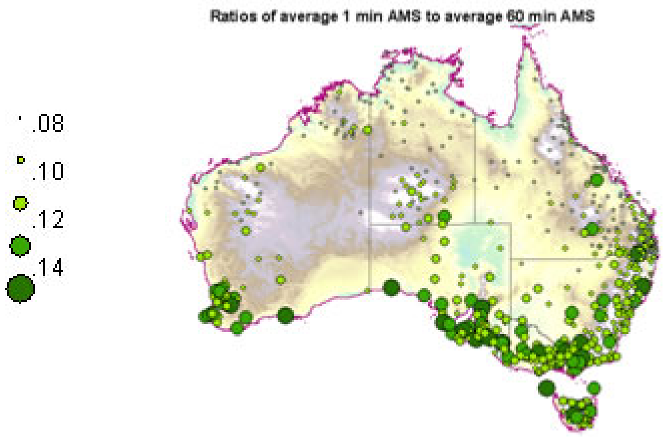

Figure 1 Ratios of average 1-minute annual maxima to average 60-minute annual maxima.

2.2. Ratios of quantiles

The Generalised Extreme Values (GEV) distribution was fitted to the AM for the 1-, 5-,10-, 15-, 30-, 60- minute durations using L-moments to estimate the parameters. Ratios of the site *n*-minute to 60-minute quantiles were calculated. The ratios did not vary much across ARI so they were averaged over ARI. A similar latitudinal variation as in Figure 1 occurs for the one-minute duration with ratios being smaller in the tropics (Figure 2).

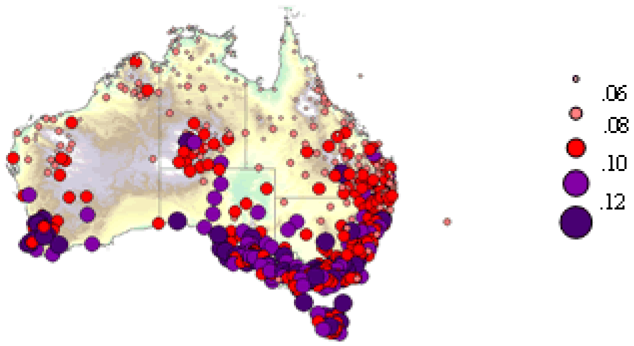

Figure 2 Ratios of 1-minute to 60-minute quantiles.

2.3. Simple Scaling

Menabde *et al* (1999) proposed (and verified) that random variables I_d have the scaling property:

$$I_d = \left(\frac{d}{D}\right)^{-\eta} I_D \qquad (1)$$

where equality is understood in the sense of equality of probability distributions, D is the longer duration, d is the shorter duration, and η is the scaling exponent.

By raising both sides of equation (1) to some power q and taking the ensemble average gives equation (2).

$$\left\langle I_d^q \right\rangle = f(q) d^{-\eta q} \qquad (2)$$

where f(q) is some function of q.

Menabde *et al* (1999) applied the method to rainfall intensity data for Melbourne, Australia and Warmbaths, South Africa. The simple scaling model proposed by Menabde *et al* (1999) has been applied to the state of Michigan, USA by Gerold and Watkins (2005) with a partial duration series approach, and Nhat *et al* (2008) to four sites located in Japan, Australia, Korea and Malaysia. Nhat *et al* (2008) suggest a break at the 1-hour duration. This may be due to very short duration rainfall being part of slightly larger duration rainfall events; for example, the 1-minute maximum may be more likely to form part of a 1-hour event than a 6-hour event in some locations. In this study, a break at the one-hour duration was evident at some tropical sites.

For the one-minute duration, the simple scaling technique was applied using data up to 15, 60 and 360 minutes. For the 5-, 10-, 15, 30-minute durations, the analysis was performed on data up to 60 and 360 minutes. The optimum result would then be determined in the verification analysis.

The annual maximum rainfall intensity series for all TBRG sites with 8 years or more of annual maxima was used and the q^{th} moments calculated and plotted for durations from 1 minute to 15 minutes, 1 minute to 60 minutes and 1 minute to 360 minutes (Figure 3 shows the moments from 1 to 360 minutes). Using pluviometer data q^{th} moments were calculated for durations of 5, 10, 15 and 30 minutes for durations up to 60 and 360 minutes. The linear fit is quite good. R^2 ranged from .90 to .98 for the one-minute data. Using durations up to 15 minutes gave improved R^2 from .95 to .99.

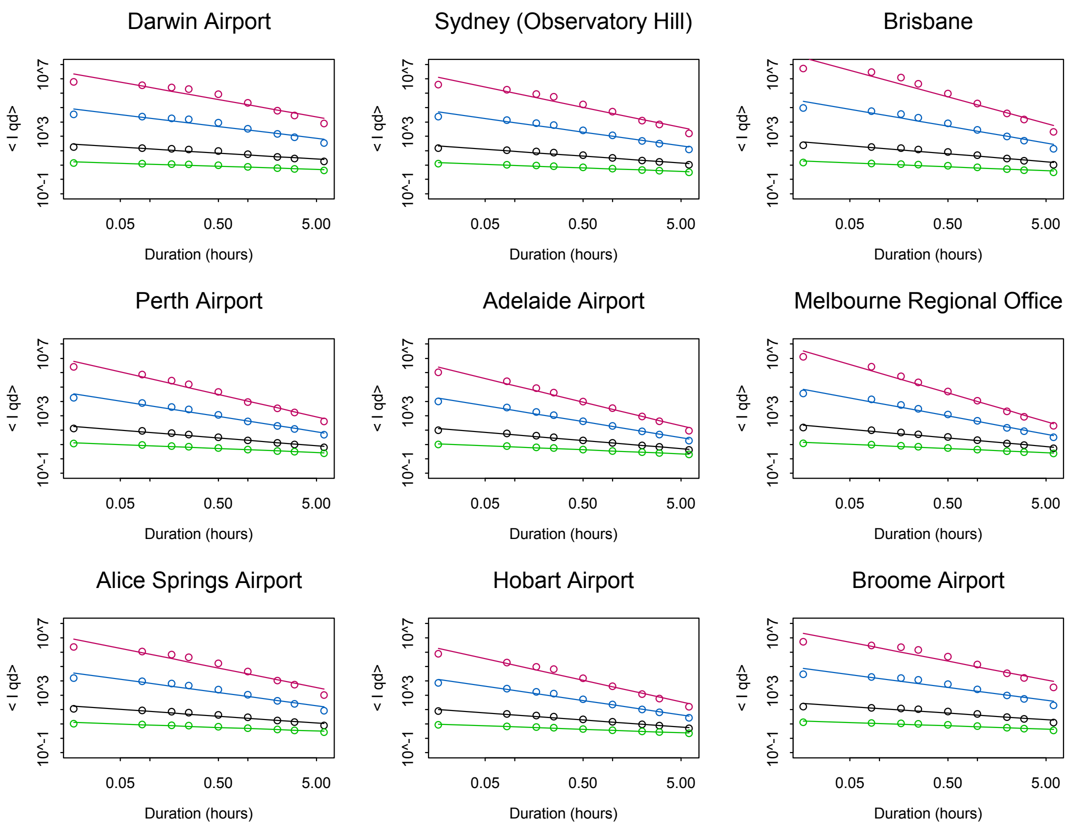

Figure 3 Scaling of the moments of the annual maxima for selected sites for durations from 1 minute to 6 hours. q=0.5: green, q=1: black, q=2: blue, q=3: red.

Using the linear relationship between $\left\langle I_d^q \right\rangle$ and *d,* the scaling exponent *–ηq* was plotted versus q (Figure 4) for the 1-minute duration to 6 hours). Once again the relationship is linear suggesting that simple scaling is applicable. If the plot was not linear, multiscaling would be more appropriate. Linear plots resulted from plots for all duration ranges and durations. The scaling parameter *η* could then be calculated using equation (1). The scaling parameter *η* does not vary greatly with distance, therefore once a regional estimate of *η* is available, equation (1) can be used to derive rainfall intensities at the shorter durations from the longer durations.

2.4. Bayesian Generalised Least Squares Regression (BGLSR)

The BGLSR (Haddad *et al*, 2010) has been applied to produce pseudo data at the 1-hour duration at daily sites. The technique involves deriving a correlation matrix and fitting a regression equation using predictors such as elevation, slope, aspect, distance from the coast to infer L-moments at the sub-daily durations.

The BGLSR has been extended down to the one-minute duration to infer L-moments at the standard sub-hour durations from the 60-minute duration using Bureau of Meteorology 60-minute pluviometer data. Site quantiles were then estimated from the L-moments through a GEV distribution.

3. VALIDATION

The four methods for deducing sub-hourly estimates were validated by comparing the statistics of the estimates with statistics from the sub-hourly pluviometer data.

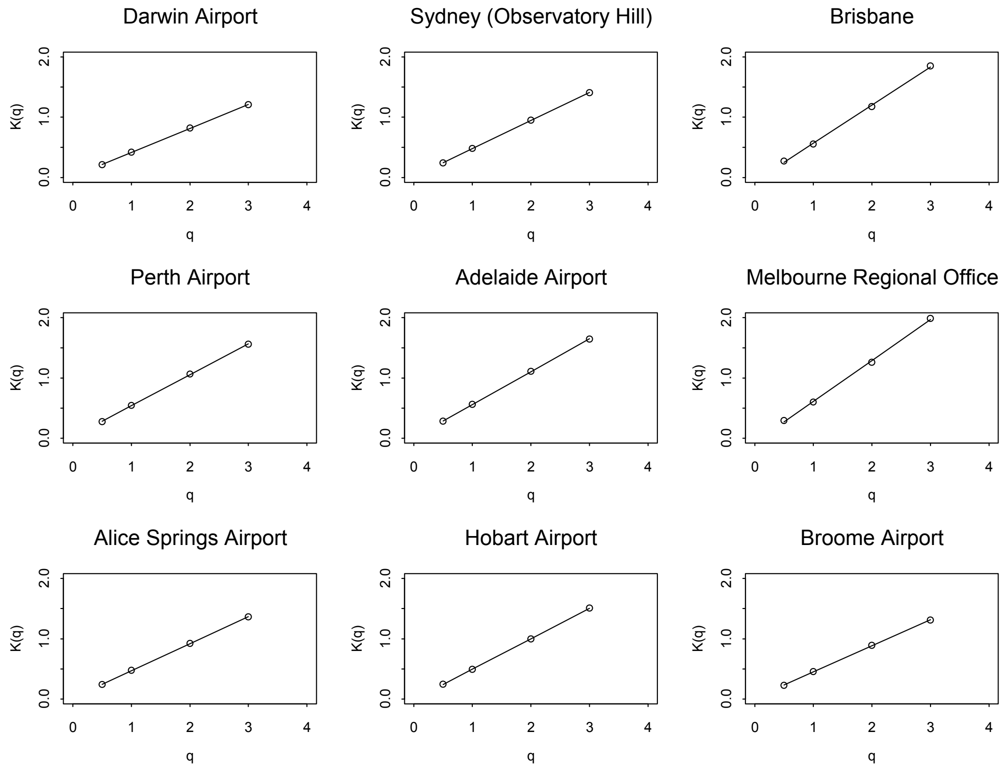

Figure 4 Scaling exponent using data from 1 minute to 360 minutes.

For ratios of AM, ratios of quantiles and simple scaling, regional estimates of ratios of AM, ratios of quantiles and η were required. Different methods were trialled by using averages of ratios of AM, ratios of quantiles and η within a circle of varying radii (e.g. 25, 50, 100, 150, 200, 400 km radii) and by averaging the number of closest sites (5, 10, 15, 20) but excluding the target site. The ratios and η displayed very little variation, so it was decided to average using the nearest five sites.

The regional average AM and quantile ratios were applied to the 60-minute site quantiles to infer quantile estimates at the standard sub-hourly durations. The regional average η_d (a different η resulted depending on the duration limits – 15, 60 and 360 minutes for the one-minute duration, and 60 and 360 minutes for the other durations) was used to estimate the sub-hourly quantiles.

The cumulative distribution functions for the sub-hourly annual maxima and the sub-hourly quantile estimates from the four methods was calculated and the Kolmogorov-Smirnov 'D' statistic used to validate the methods. This test compares the cumulative distribution function (CDF) of the modeled data with that of the actual data.

The 'D' statistic for the one-minute duration is shown in Figure 5. At the one minute duration, D is smaller for ratios of quantiles and AMS and simple scaling using data up to 60 minutes. Simple scaling using data to 15 and 360 minutes and GLSR do not perform as well.

In order for the 'D' statistic to be reliable, the CDF for the data needs to be reliable also. This is best achieved when record lengths are long. Very short record lengths do not have a reliable CDF to compare with. For the longer sub-hourly durations the results from data with sites having at least 40 years of data is shown in Figure 6.

For the 5-, 10-, 15-, 30- minute durations, simple scaling using data to 360 minutes gave better results than using data to 60 minutes. The performance of simple scaling depends on the duration limit used.

This suggests that there may be an optimum duration limit for each duration and that by calibrating the method for the duration of interest, the simple scaling results could be improved. It is possible that simple scaling using data up to some intermediate duration between 60 and 360 minutes could give better results.

Overall, ratios of AMS gives slightly improved values compared with the other methods. Using ratios of AMS also has the advantage of ease of application.

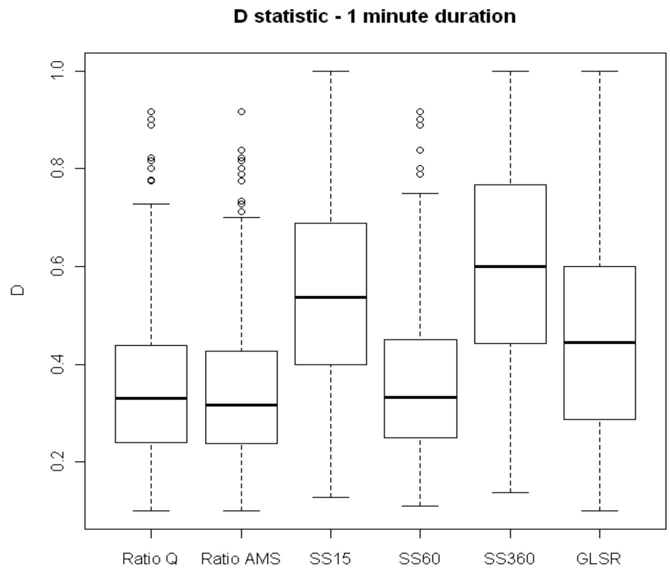

Figure 5 'D' statistic for the one-minute duration.

4. SUMMARY

Four methods to estimate sub-hourly rainfall from 60-minute data have been explored. The methods are: ratios of AM, ratios of quantiles, simple scaling and BGLSR.

TBRG data recording rainfall at a resolution of 1 minute has been used for the 1-minute duration. For the 5-, 10-, 15-, 30-minute durations, Dines pluviograph data as well as TBRG has been used.

At the very short 1-minute duration ratios of AM and ratios of quantiles show a latitudinal variation with ratios being smaller in the north than in the south of Australia.

Simple scaling has been applied using data up to 15, 60 and 360 minutes for the 1-minute duration. For the longer subhourly durations, data up to 60 and 360 minutes was used.

The Kolmogorov-Smirnov test has been used to validate the methods. At the 1-minute duration simple scaling performed best using data up to 60 minutes. At the longer subhourly durations simple scaling performed best using data up to 360 minutes. At the 1-minute duration, ratios of AM, ratios of quantiles and simple scaling using data up to 60 minutes perform similarly with ratios of AM being marginally best. At the longer subhourly durations, ratios of quantiles, ratios of AM and simple scaling using data up to 360 minutes perform similarly. Overall, using ratios of AM gave the best result.

5. ACKNOWLEDGEMENTS

The helpful suggestions of Dr. Fiona Johnson, Dr. Robert Smalley, Dr. Grace Mitchell and Dr. Prasantha Hapuarachchi are gratefully acknowledged.

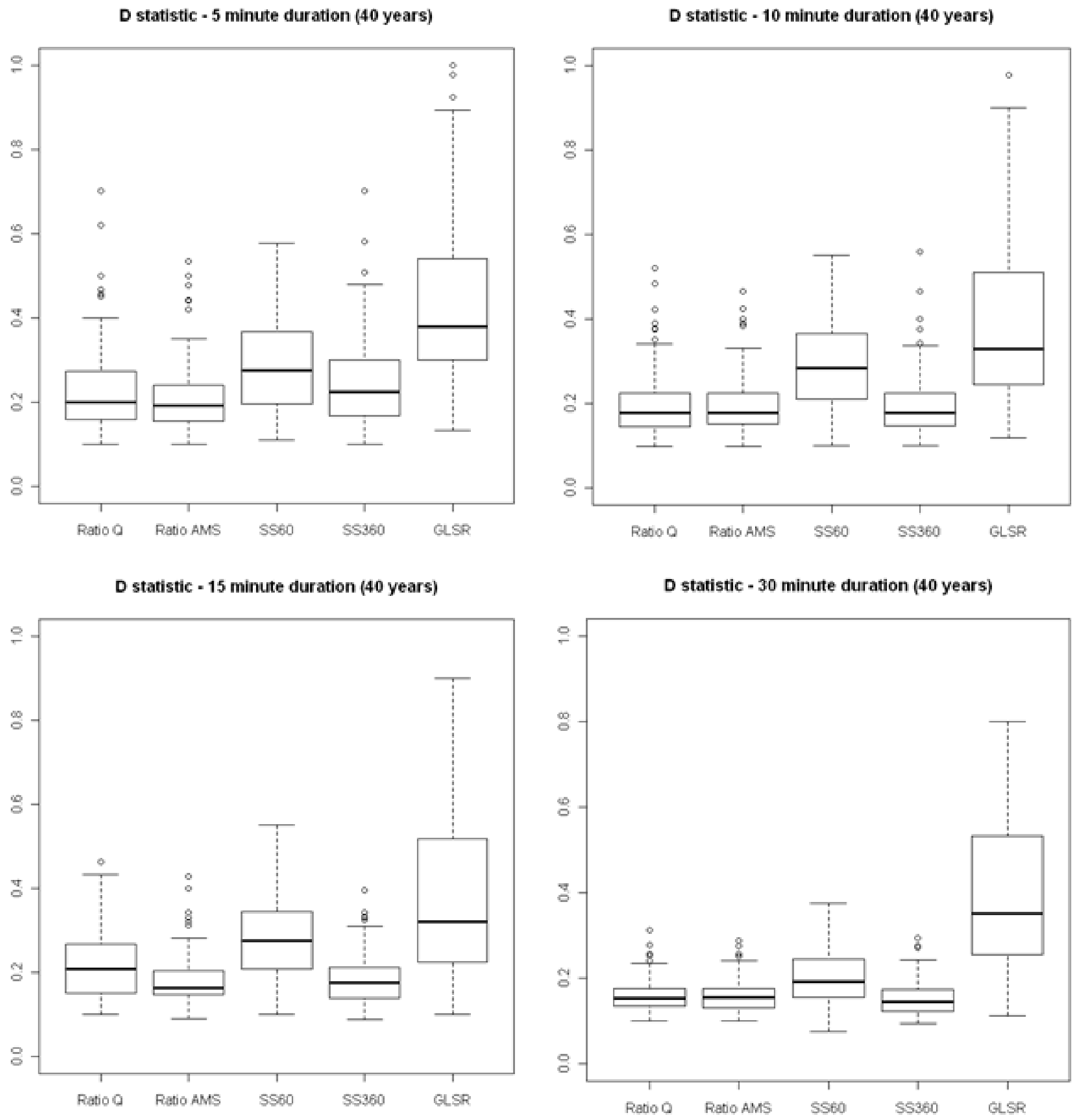

Figure 6 'D' statistic for the 5-,10-,15- and 30-minute durations using 40-year datasets.

6. REFERENCES

Bonnin, G M, D Martin, B Lin, T Parzybok, M Yekta, and D Riley. (2004, revised 2006). *Precipitation-Frequency Atlas of the United States: Semiarid Southwest (Arizona, Southeast California, Nevada, New Mexico, Utah).* NOAA Atlas 14, Volume 1, Version 4.0.

Gerold, L A and D W Watkins Jr. (2005). *Short Duration Rainfall Frequency Analysis in Michigan using Scale-Invariance Assumptions*. Journal of Hydrologic Engineering, 450-457.

Haddad, K, A Rahman, J Green. (2010). *Design rainfall estimation in Australia: a case study using L moments and Generalized Least Squares Regression*. Stochastic Environmental Research and Risk Assessment, 30 November 2010.

Mandapaka, P.V, P. Lewandowski, W. E. Eichinger, and W. F. Krajewski. (2009). *Multiscaling analysis of high resolution space-time lidar-rainfall*. Nonlinear Processes in Geophysics, 16, 579–586.

Menabde, M, A Seed, G Pegram. (1999). *A simple scaling model for extreme rainfall*. Water Resources Research, 35, pages 335 – 339.

Nhat LeMinh, Tachikawa Y, Sayama T and Takara K (2008). *Estimation of Sub-hourly and Hourly IDF Curves using Scaling Properties of Rainfall at Gauged Site in Asian Pacific Region*. Annuals of Disas. Prev.Res. Inst., Kyoto Univ., No. 51 B.

Perica, S Dietz, S Heim, L Hiner, K Maitaria, D Martin, S Pavlovic, I Roy, C Trypaluk, D Unruh, F Yan, M Yekta, T Zhao, G Bonnin, D Brewer, L Chen, T Parzybok, J Yarchoan. (2011). *Precipitation-Frequency Atlas of the United States: California.* NOAA Atlas 14. Volume 6, Version 2.